RenR 480/711

Non-parametric statistics

What can we do if assumptions of T-Tests & Anova are not met?

Non-parametric tests do not assume:

- Normality
- Equal variances
- No large outliers

Important is, however: (as for all statistical tests)

- Independence of samples
- Replication
- Randomization

Rank based tests

How do rank based tests work?

1. Rank the values of observation: 1 to n
2. Calculate the sum of ranks for groups/treatments
3. Calculate differences between sum of ranks among groups/treatments
   →If the difference in sum of ranks is 0, there is no effect
4. Build a probability distribution based on all possible combinations of ranks
5. Derive a p-value from the probability distribution based on the difference in rank sum

$$H_0\ Null\ Hypothesis:\ Ranksum_A - Ranksum_B = 0$$

Rank based tests

Example

Variety	Yield	Rank
A	720	2
A	740	1
B	515	3
B	480	4

$\Sigma_A = 3$

$\Sigma_B = 7$

$\Sigma_A - \Sigma_B = -4$

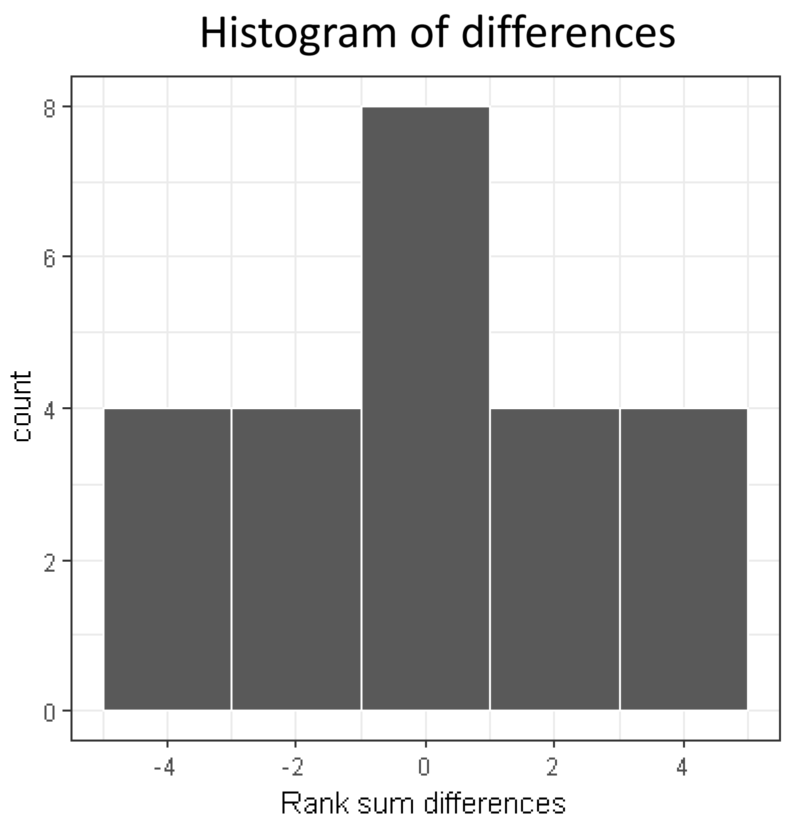
Rank sum difference

Rank based tests

Possible ranking combinations (4 ranks): !4 = 4*3*2*1=24

Nr	Rank1_A	Rank2_A	Rank3_B	Rank4_B	Ranksum_A	Ranksum_B	Rank-sum difference
1	1	2	3	4	3	7	-4
2	1	2	4	3	3	7	-4
3	1	3	2	4	4	6	-2
4	1	3	4	2	4	6	-2
5	1	4	2	3	5	5	0
6	1	4	3	2	5	5	0
7	2	1	3	4	3	7	-4
8	2	1	4	3	3	7	-4
9	2	3	1	4	5	5	0
10	2	3	4	1	5	5	0
11	2	4	1	3	6	4	2
12	2	4	3	1	6	4	2
13	3	1	2	4	4	6	-2
14	3	1	4	2	4	6	-2
15	3	2	1	4	5	5	0
16	3	2	4	1	5	5	0
17	3	4	1	2	7	3	4
18	3	4	2	1	7	3	4
19	4	1	2	3	5	5	0
20	4	1	3	2	5	5	0
21	4	2	1	3	6	4	2
22	4	2	3	1	6	4	2
23	4	3	1	2	7	3	4
24	4	3	2	1	7	3	4

Rank based tests



Probability of each difference occurring:

-4: 4 out of 24 times
-2: 4 out of 24 times
0: 8 out of 24 times
2: 4 out of 24 times
4: 4 out of 24 times

H_0 *Null Hypothesis*: $Ranksum_A$- $Ranksum_B$ $= 0$

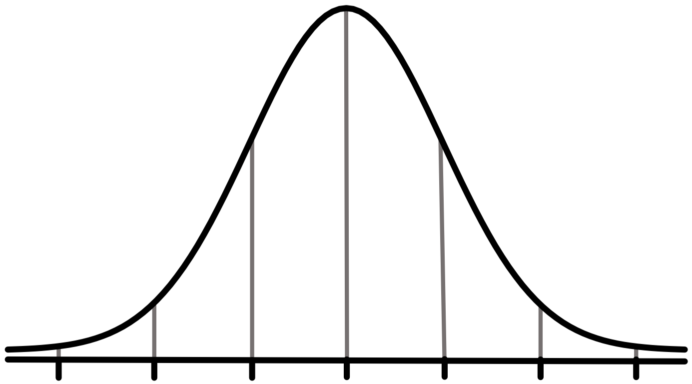
Our observed difference: -4

p-value: -4/24 = 0.166: not significant

More samples = more bins in the histogram = closer to normal distribution

Rank based tests

Rank sum differences are normally distributed!



Wilcoxon-Rank-sum tests

→ Non-parametric alternatives to the T-Test

→ These tests assume similar distributions within groups

Wilcoxon-Signed-Rank-Test (alternative to a One-sample T-Test)

```
wilcox.test(sample, mu=-5, alternative="greater")
```

Wilcoxon-Mann-Whitney-Test (alternative to a Two-sample T-Test)

```
wilcox.test(sample_A, sample_B, alternative="two.sided")
```

A paired test is also possible:

```
wilcox.test(Before, After, alternative="two.sided", paired=T)
```

In R, p-values are approximated, but you can force exact p-values:

```
wilcox.test(sample_A, sample_B, alternative="two.sided“, exact=T)
```

This means that all possible combinations of ranks are calculated.

With large sample sizes, computing time may be an issue.

Kolmogorov-Smirnov Test

→ Non-parametric alternative to the T-Test

→ This test can handle different distributions within groups

→ Two sample test only

Kolmogorov-Smirnov Test (alternative to a Two-sample T-Test)

```
ks.test(sample_A, sample_B, alternative="greater")
ks.test(sample_A, sample_B, alternative="two.sided")
ks.test(before, after, alternative="two.sided", paired=T)
```

→Less powerful test than Wilcoxon

Kruskal-Wallis Test

→Non-parametric alternative to a One-way-Anova

In R:

```
kruskal.test(Response~Treatment)
```

→ A significant effect indicates a difference in medians among groups

→ In case of significant effects: Pairwise Wilcoxon-Mann-Whitney-Tests

→ Follow up with the p.adjust function to protect against Type I error

```
p.adjust(p_values, method="holm")
```

Friedman's Rank Test

→ Non-parametric alternative to a Two-way-Anova

→ Alternative to analyse randomized complete block designs if assumptions for Anova are not met

```
friedman.test(Respones~Treatment|Block)
```

→ In case of significant effects: Pairwise Wilcoxon-Mann-Whitney-Tests

→ Follow up with the p.adjust function to protect against Type I error