

RenR 480/711

Standard error, Normal & t-distribution,
confidence intervals, z-scores and t-values

Recap

Descriptive statistics

→ summary of data (graphical and/or numerical)

Distribution

→ Probability associated with each possible value of a variable

Parameter

→ Population characteristic

Statistic

→ Estimate of parameter using a sample

Recap

Bias

→ systematic deviation of statistic from parameter

Error

→ random deviation of a statistic from parameter

Inferential statistics

→ making predictions, drawing conclusions from data that is subject to random variation

Standard error

→ standard deviation of any statistic

New Vocabulary

New:

- Alpha level

Predetermined probability at which we make a decision

- P-value

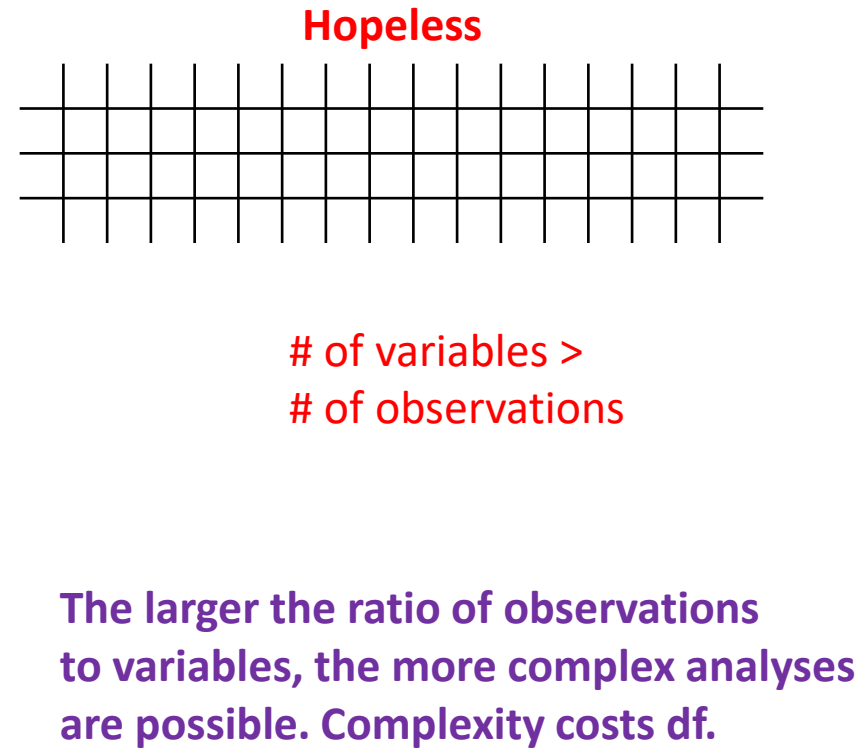
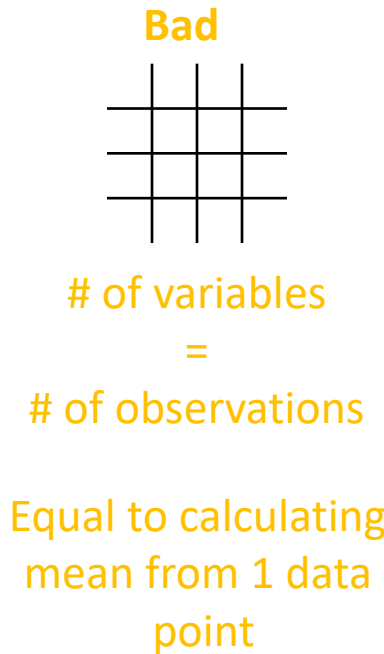
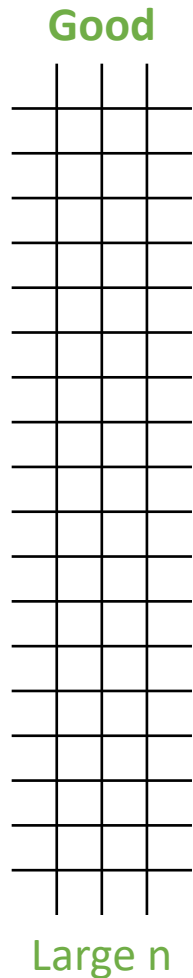
The probability that an observed difference is due to random chance

Degrees of freedom

The number of observations in your data that are free to vary when calculating a statistic

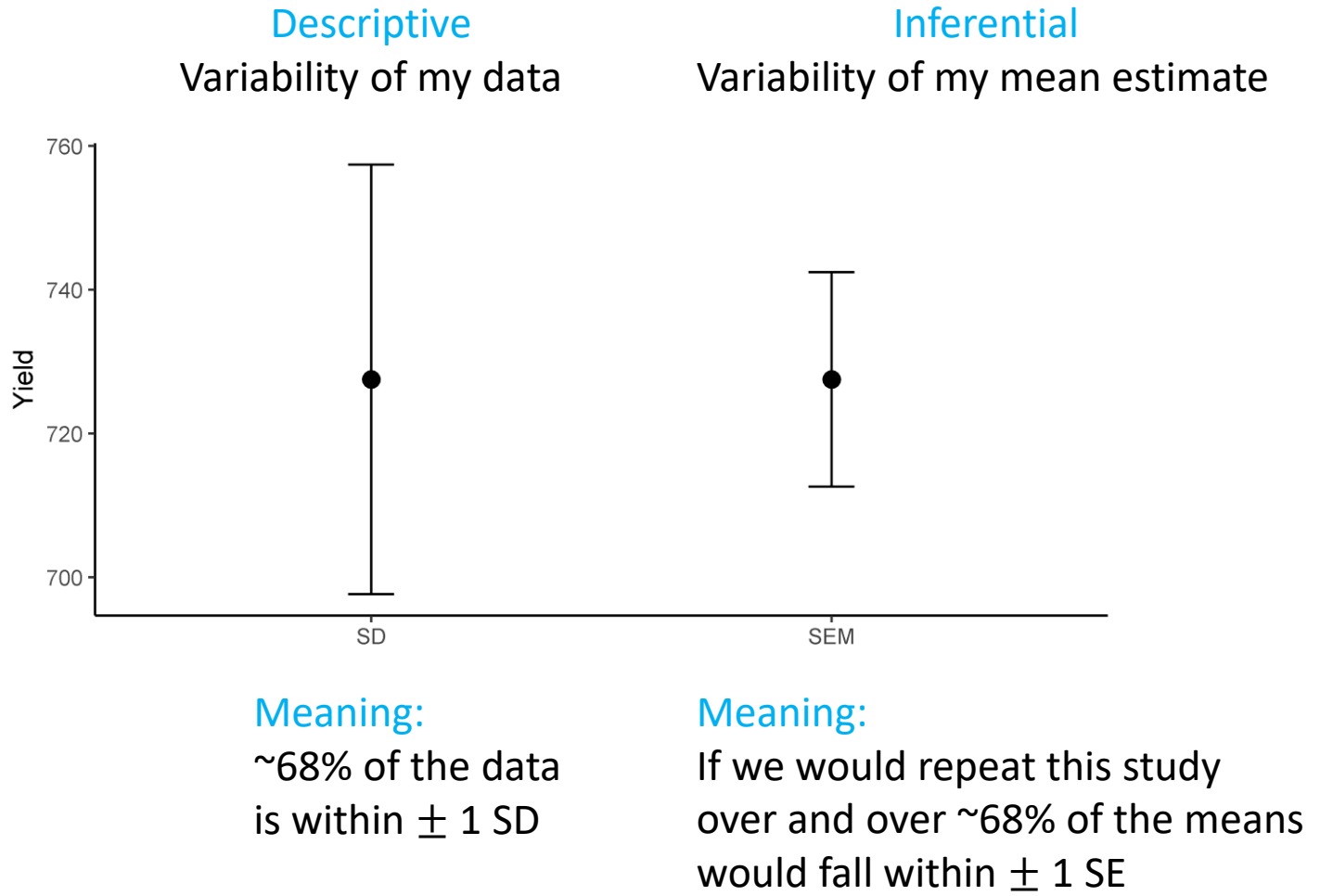
Excursion: Degrees of freedom

Degrees of freedom are good!



Intro to inferential statistics

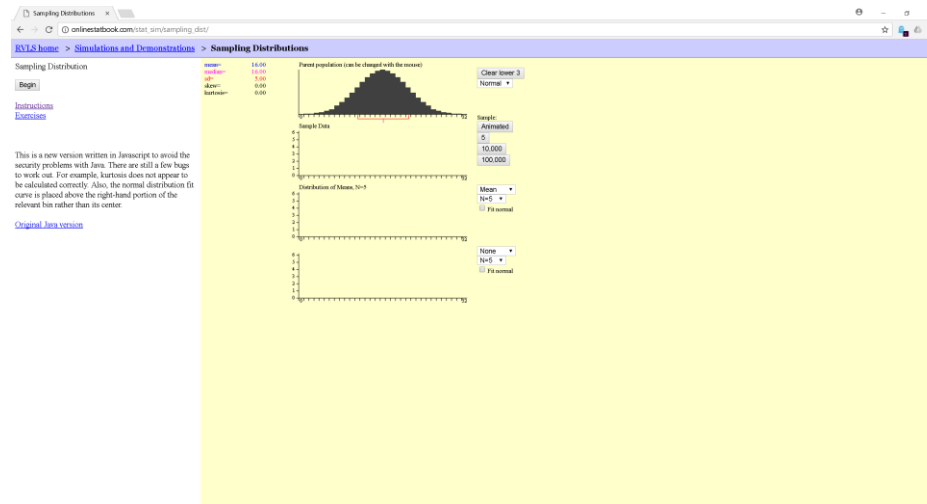
What's the difference between standard deviation and standard error?



Central limit theorem

Please open on in our browser:

www.tinyurl.com/clt-simulator



When does the central limit theorem fail?

Another simulation: Standard error of the mean

Please enter the following code:

```
s1 = rnorm(10, mean=15, sd=5)
```

```
mean(s1)
```

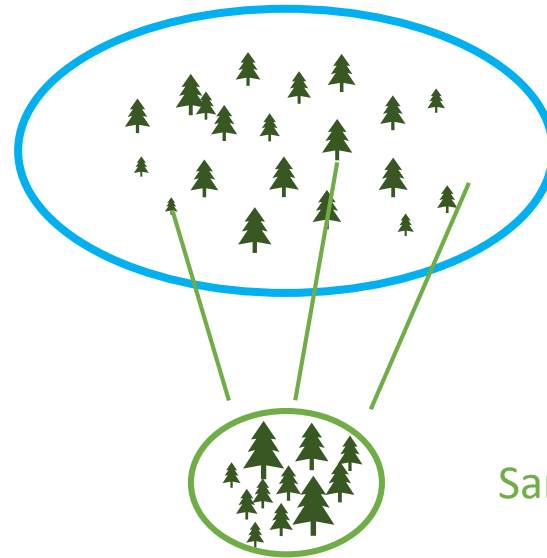
```
sd(s1)
```

```
sd(c(...))
```

```
se1 = sd(s1)/sqrt(10)
```

or

```
se1 = your s/sqrt(10)
```



Population

$$\mu = 15\text{m}$$
$$\sigma = 5\text{m}$$

Sample of 10

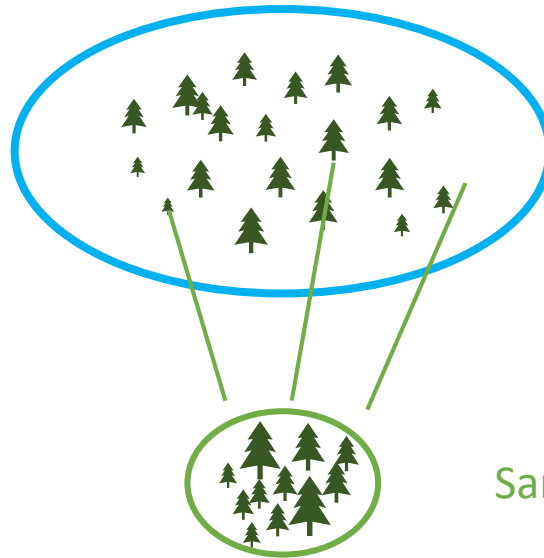
$$\bar{x} = ?$$

$$s = ?$$

$$SEM_{\bar{x}} = \frac{s}{\sqrt{n}}$$

Standard error of the mean

$$SEM_{\bar{x}} = \frac{s}{\sqrt{n}}$$



Population

$$\mu = 15\text{m}$$

$$\sigma = 5\text{m}$$

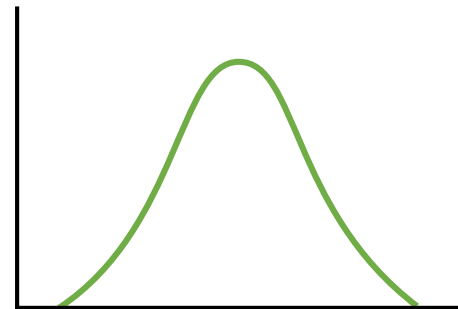
Sample of 10

$$\bar{x} = 15.2\text{m}$$

$$s = 4.4$$

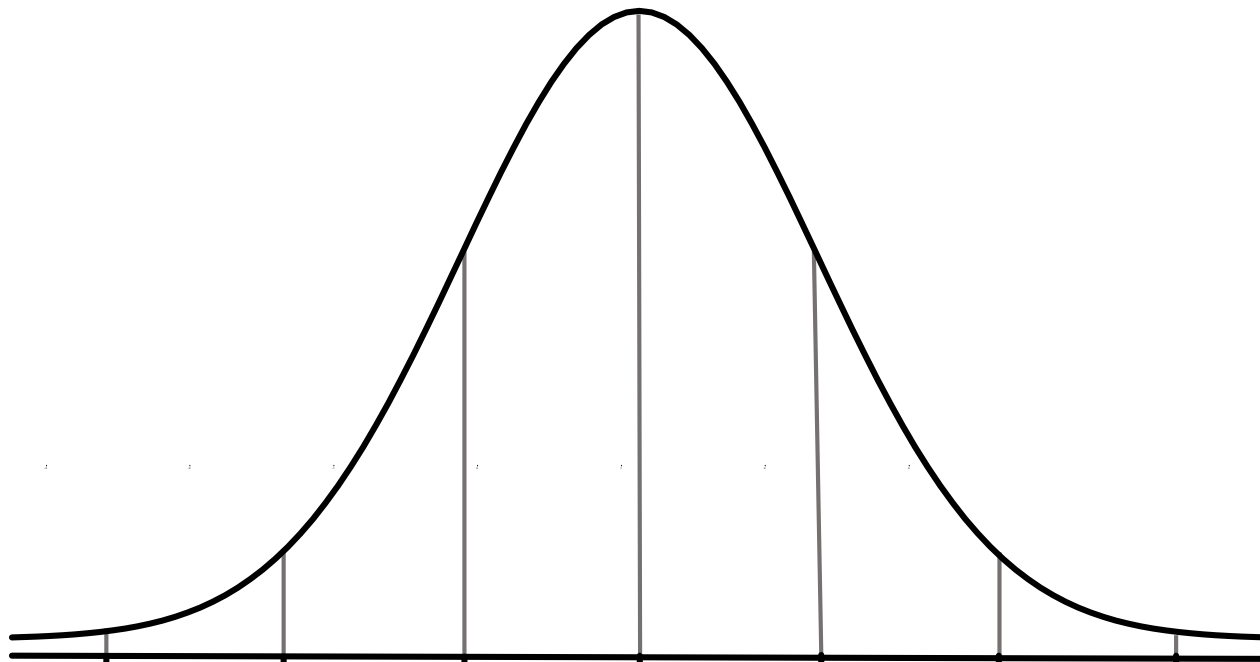
By repeated sampling what do we get?

A distribution of \bar{x} ! ... and it's normal!



Normal distribution

Because we know that sample means are normally distributed, we can now make assumptions based on it.



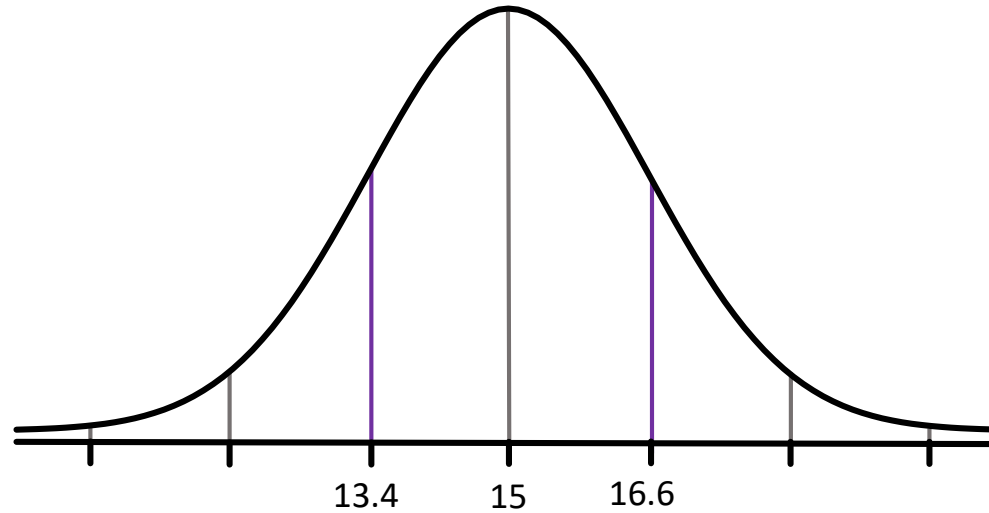
Standard deviation	-3	-2	-1	0	1	2	3
Percentile	0.1%	2.3%	15.9%	50%	84.1%	97.7%	99.9%

Standard errors & Confidence intervals



$$\bar{x} = 15$$

$$SEM_{\bar{x}} = 1.6$$



Distribution of
the means

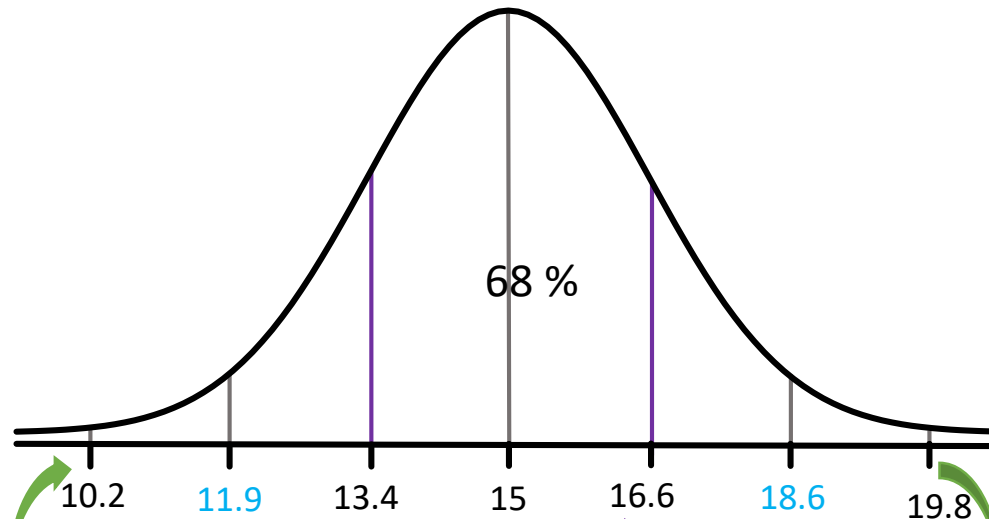
Original values

Standard errors & Confidence intervals



$$\bar{x} = 15$$

$$SEM_{\bar{x}} = 1.6$$



Distribution of the means

Original values

$$\text{value} = \text{z-score} * \text{SEM} + \bar{x}$$

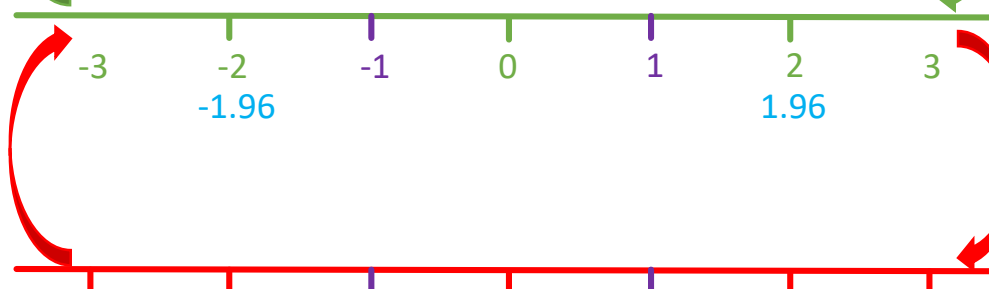
$$\text{z-score} = (\text{value} - \bar{x}) / \text{SEM}$$



Standard errors (z-scores)

$$\text{z-score} = \text{qnorm}(\text{p-value})$$

$$\text{p-value} = \text{pnorm}(\text{z-score})$$



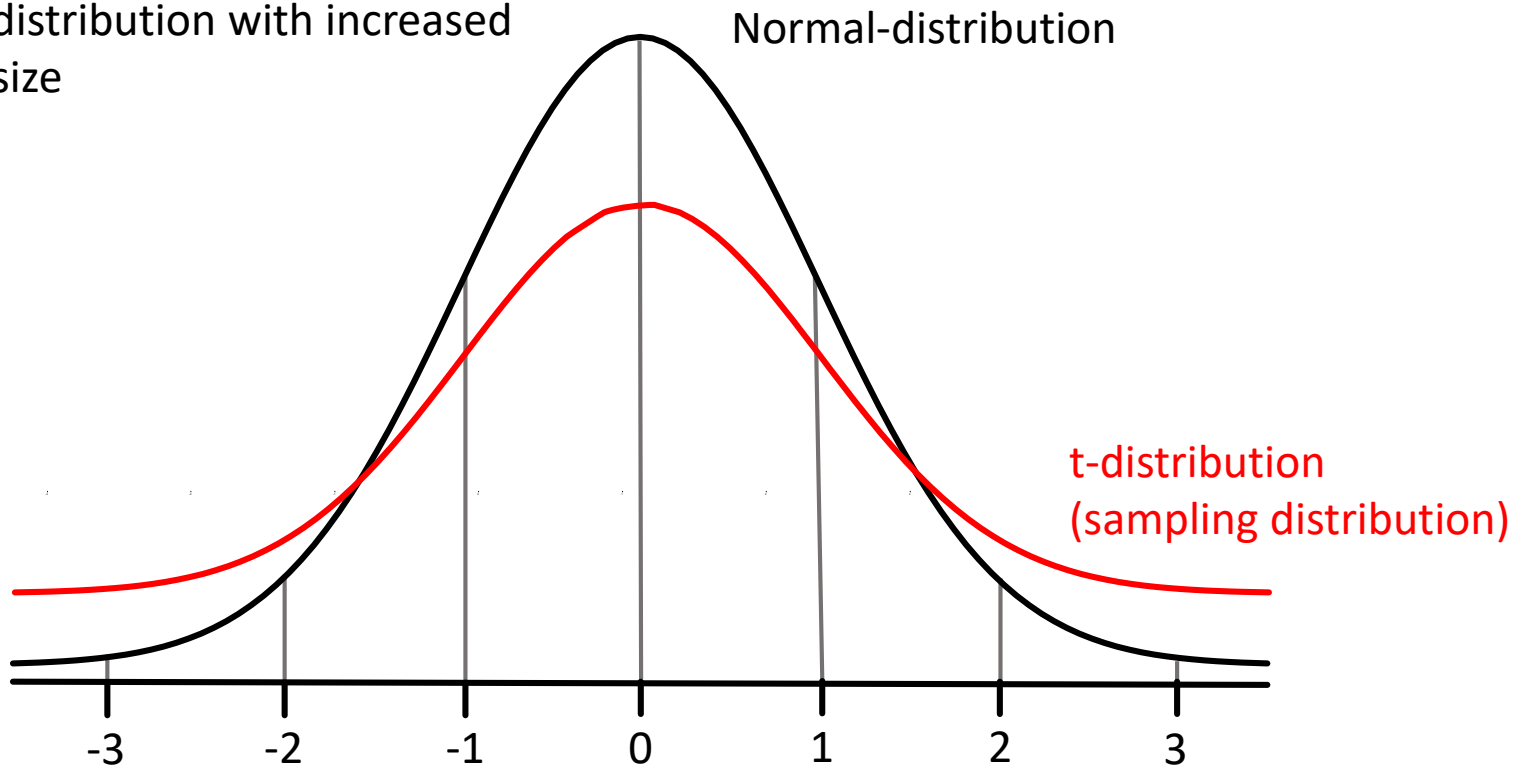
Percentiles (p-values)

95% Confidence interval = (11.9, 18.6)

T-distribution

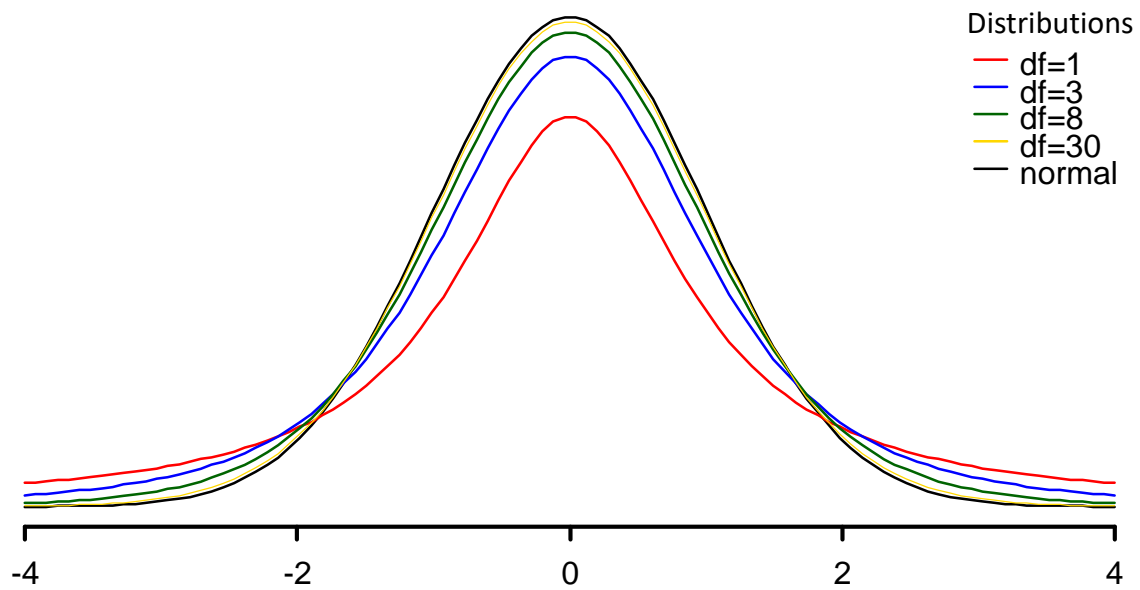
Features:

- Wider tails
- Degrees of freedom: $n-1$
- Approaches normal the shape of a normal distribution with increased sample size



Fixes biased sample means when sample size is low

T-distribution



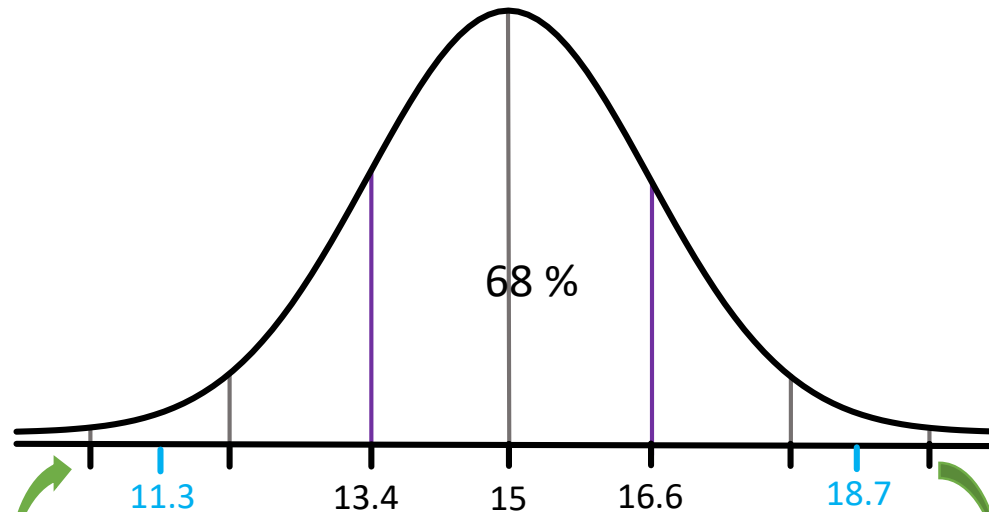
T-distribution



$$\bar{x} = 15$$

$$SEM_{\bar{x}} = 1.6$$

$$n=10$$



Original values

$$\text{value} = t\text{-value} * SEM + \bar{x}$$

$$t\text{-value} = (\text{value} - \bar{x}) / SEM$$



t-values
(standard error)

$$t\text{-value} = qt(p\text{-value}, df)$$

$$p\text{-value} = pt(t\text{-value}, df)$$



Percentiles
(p-values)

95% Confidence interval = (11.3, 18.7)

Challenge

Lentil varieties on Farm 1

Can you calculate the 90% CI for Var A, B & C?

Enter the data like this:

VarA = c(720, 740, 690, 760)

VarB = c(515, 480, 545, 492)

VarC = c(540, 502, 510, 505)

FARM	VARIETY	YIELD
Farm1	A	720
Farm1	A	740
Farm1	A	690
Farm1	A	760
Farm1	B	515
Farm1	B	480
Farm1	B	545
Farm1	B	492
Farm1	C	540
Farm1	C	502
Farm1	C	510
Farm1	C	505

1. Decide what p-value you need (1-0.9=0.1; 0.1/2=0.05; so either 0.05 or 0.95)

2. Calculate the t-values → t-value = qt(p-value, df)

3. Convert t-values to real values: value = t-value*SE+ \bar{x}

use $\text{VarA}_m = \text{mean}(\text{VarA})$ and $\text{SE} = \text{sd}(\text{VarA}) / \text{sqrt}(\text{length}(\text{VarA}))$ to calculate mean and standard error