

RenR 480/711

Hypothesis testing

One way Analysis of Variance (ANOVA)









Hypothesis testing

Lentil experiment

$$\bar{x}_{\text{variety A}} = 728 \text{ kg/ha}$$

$$\bar{x}_{\text{variety B}} = 514 \text{ kg/ha}$$

Farm1

H_0 Null Hypothesis: $\mu_{\text{variety A}} = \mu_{\text{variety B}}$

H_1 Alternative Hypothesis: $\mu_{\text{variety A}} \neq \mu_{\text{variety B}}$

All statistics can do is to **try to reject H_0**

Is random chance a plausible explanation for the observed difference?

$p = 0.2$ Yes, one out of 5 times

$p = 0.05$ Yes, one out of 20 times

$p = 0.0001$ Yes, one out of 10000 times

Hypothesis testing

Type I error

- The chance to reject H_0 when it is actually true
- Type I error = α
- Depends on the chosen α -level (Predetermined probability at which we make a decision)

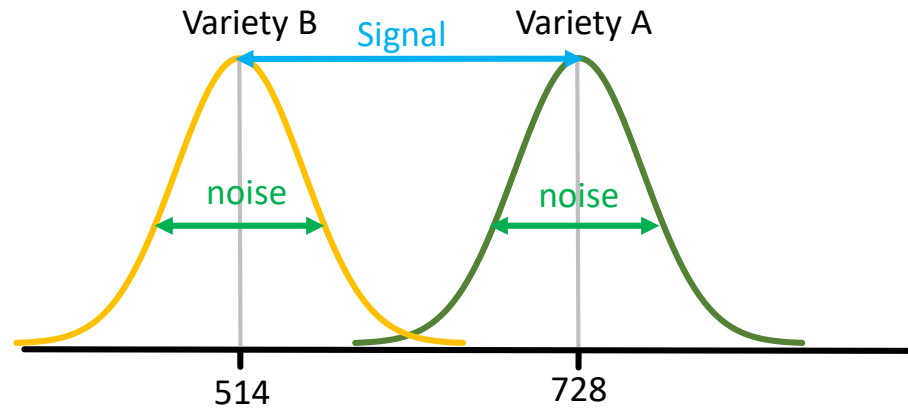
$\alpha \downarrow \begin{matrix} 0.05 \\ 0.01 \end{matrix}$ Type I error \downarrow

Type II error

- The chance to accept H_0 when it is actually false
- There is not enough evidence in the data to reject H_0

$\alpha \downarrow \begin{matrix} 0.05 \\ 0.01 \end{matrix}$ Type II error \uparrow

t-Test for two samples

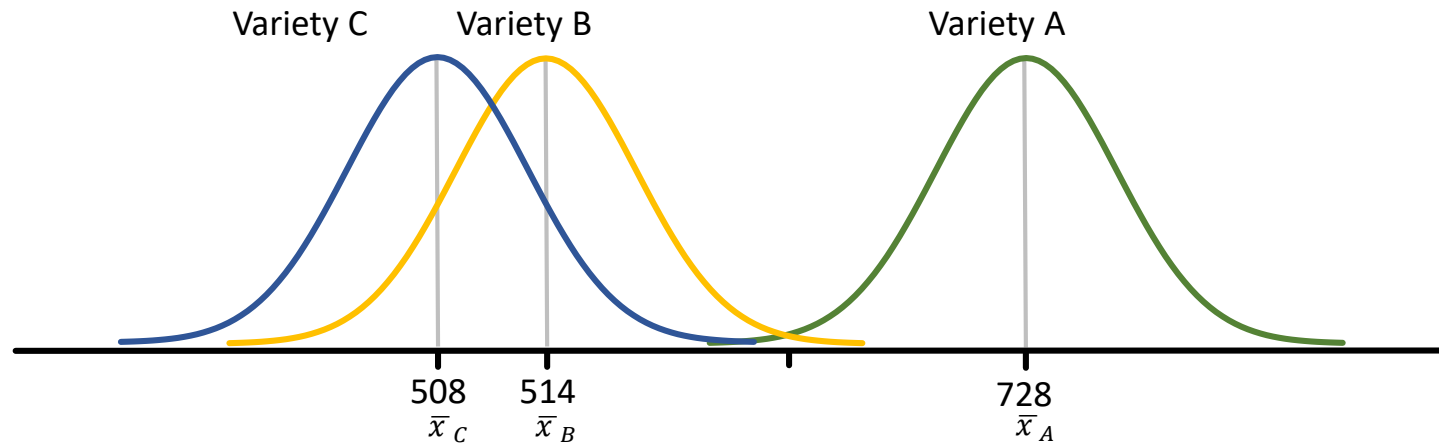


$$t = \frac{\text{signal}}{\text{noise}} \quad t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad \text{Pooled SE}$$

H_0 Null Hypothesis: $\mu_{\text{Variety A}} = \mu_{\text{Variety B}}$

One way ANOVA

For 3 or more sample groups

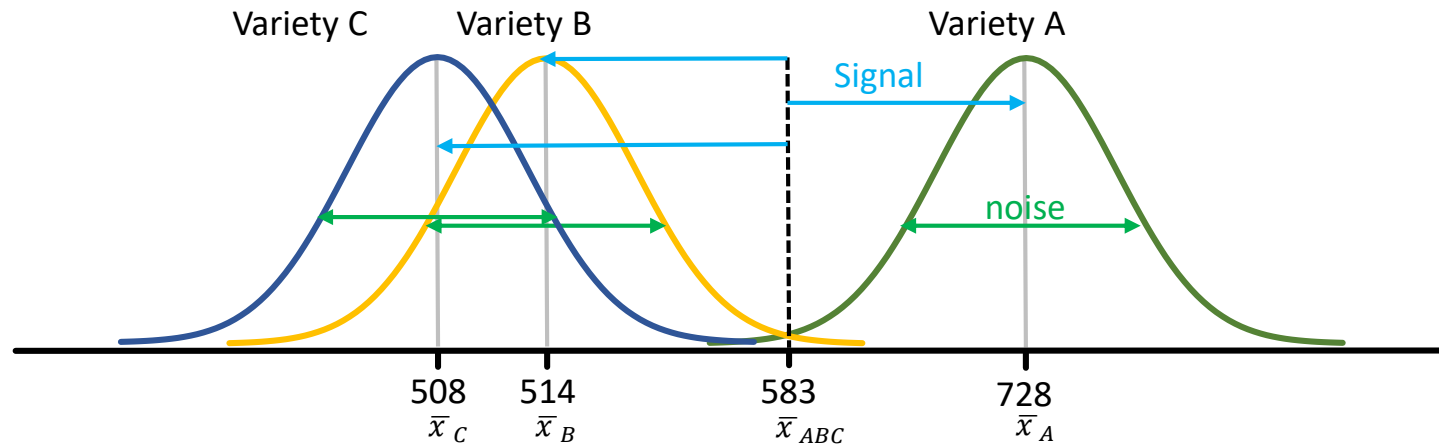


H_0 Null Hypothesis: $\mu_{\text{variety A}} = \mu_{\text{variety B}} = \mu_{\text{variety c}}$

H_0 is rejected when the probability for any difference between groups being due to chance is less than the alpha-level.

One way ANOVA

For 3 or more sample groups



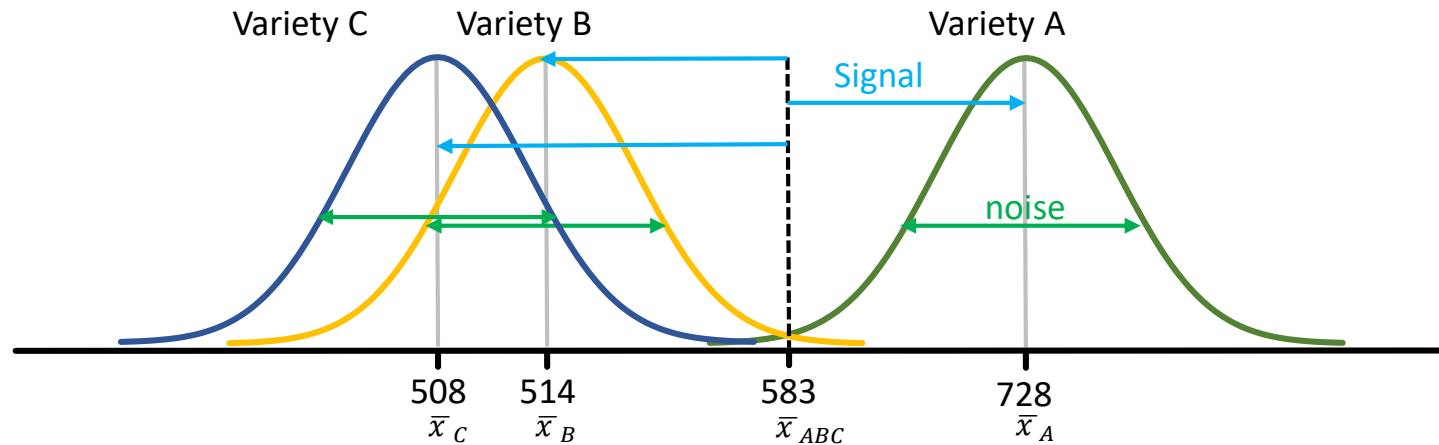
$$F = \frac{\text{signal}}{\text{noise}} = \frac{\text{variance}_{\text{between}}}{\text{variance}_{\text{within}}}$$

$$\text{variance}_{\text{between}} = \frac{\sum_i^n (\bar{x}_i - \bar{x}_{All})^2}{n - 1} \times r$$

$$\text{variance}_{\text{within}} = \frac{\sum_i^n (\text{variance}_i)}{n}$$

One way ANOVA

For 3 or more sample groups



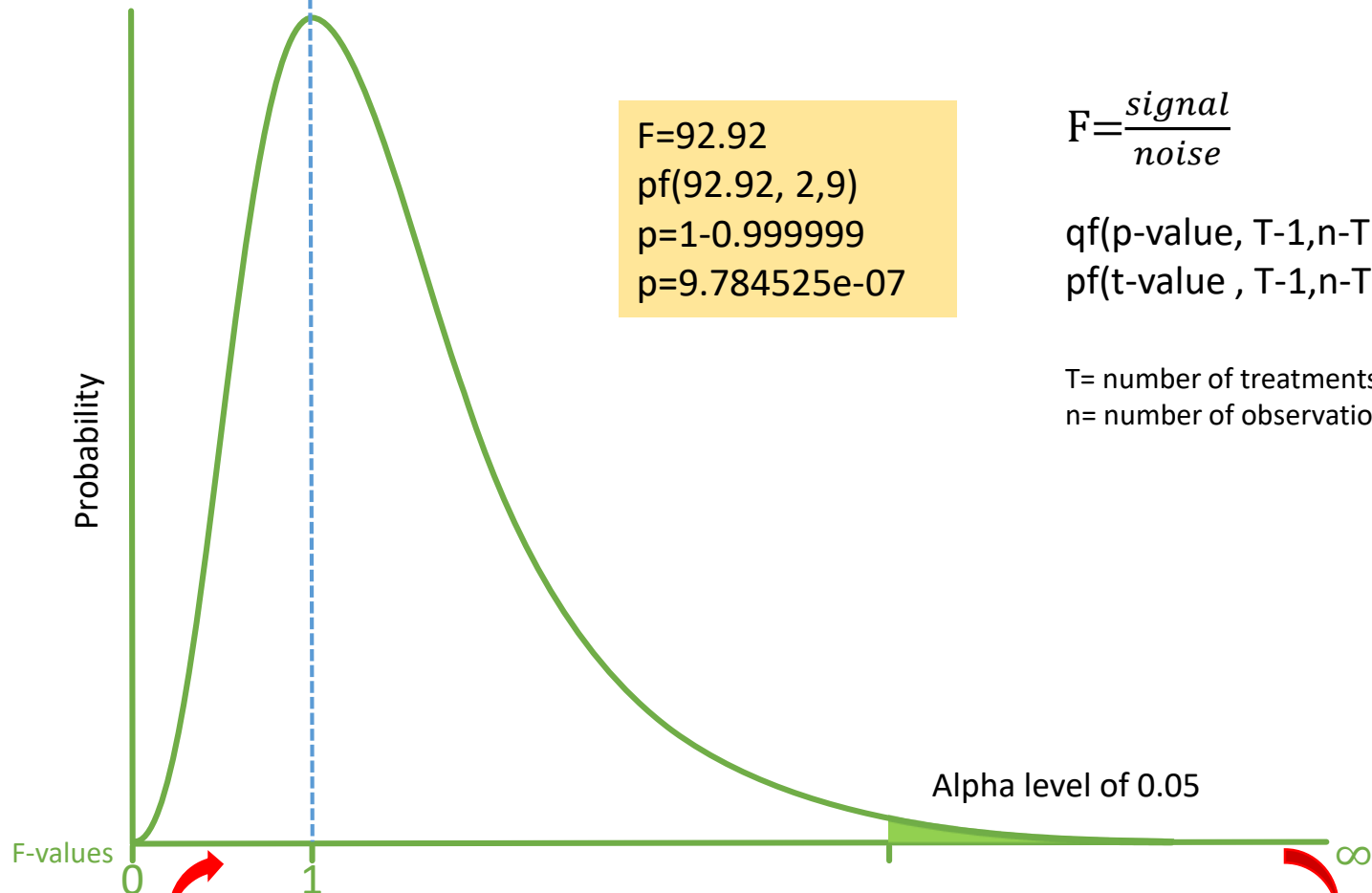
$$variance_{between} = \frac{\sum_i^n (\bar{x}_i - \bar{x}_{All})^2}{n-1} \times r = \frac{(728-583)^2 + (514-583)^2 + (508-583)^2}{3-1} \times 4 = \mathbf{62463}$$

$$variance_{within} = \frac{\sum_i^n (variance_i)}{n} = \frac{891.67 + 819.33 + 305.58}{3} = \mathbf{672}$$

$$F = \frac{signal}{noise} = \frac{variance_{between}}{variance_{within}} = \frac{62463}{672} = 92.92$$

F-distribution

Signal < Noise | *Signal > Noise*



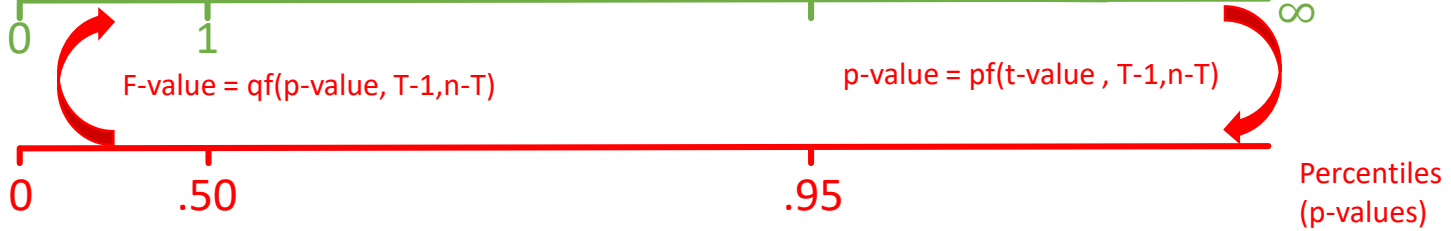
F=92.92
pf(92.92, 2,9)
p=1-0.999999
p=9.784525e-07

$$F = \frac{\text{signal}}{\text{noise}}$$

qf(p-value, T-1, n-T)
pf(t-value, T-1, n-T)

T= number of treatments
n= number of observations

Alpha level of 0.05



F-value = qf(p-value, T-1, n-T)

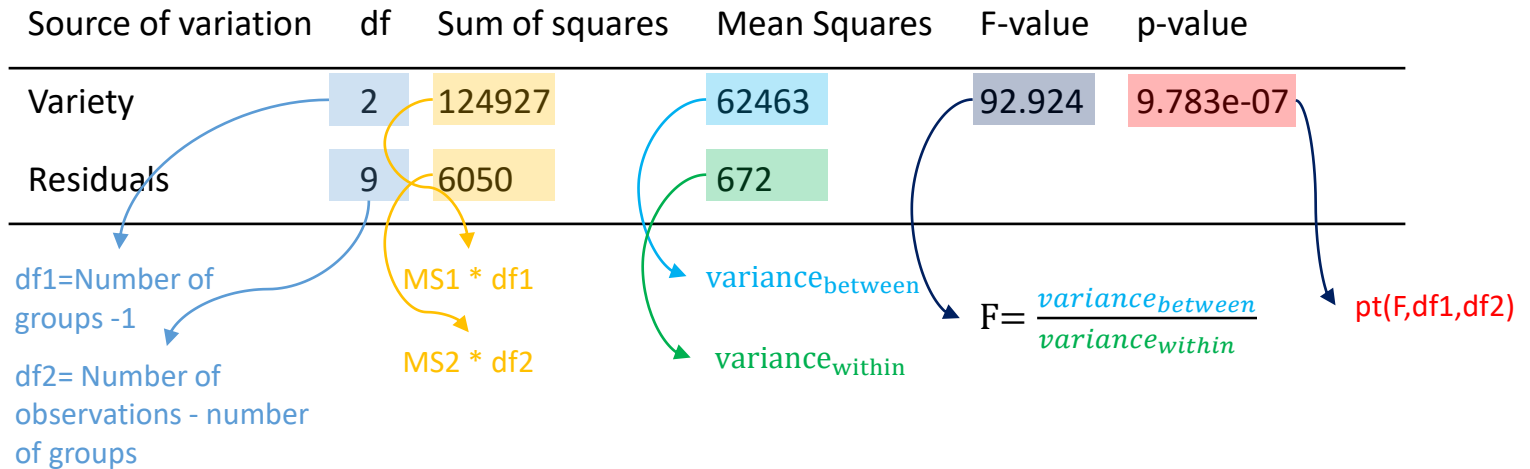
p-value = pf(t-value, T-1, n-T)

Percentiles
(p-values)

ANOVA table

```
> anova(out1)
Analysis of Variance Table

Response: YIELD
      Df Sum Sq Mean Sq F value    Pr(>F)
VARIETY  2 124927   62463   92.924 9.783e-07 ***
Residuals 9   6050    672
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



ANOVA vocabulary

Treatment

Predictor variable e.g. Variety, Fertilisation, Irrigation, Age-class, water quality-class

Treatment level

Value of predictor variable e.g. A,B,C / C,1xN,2xN / 1,2,3

Covariate

Undesired, uncontrolled predictor variable, confounding, e.g. soil moisture-class of plots

F-value

$$F = \frac{\text{signal}}{\text{noise}} = \frac{\text{variance}_{\text{between}}}{\text{variance}_{\text{within}}} = \frac{\text{mean squares}_{\text{treatment}}}{\text{mean squares}_{\text{error}}}$$

p-value

Probability that the observed difference in means is due to random chance